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THE USEFULNESS OF MATHEMATICS IN NATURAL SCIENCES-MOTIVATION TO THE TEACHING OF PHYSICS AND MATHEMATICS

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ABSTRACT.

Mathematics is studied in every school and university. Nevertheless it is rather unknown and often misunderstood. School children hence also adults, since children become adults are often afraid of mathematics or hate it. Where is the source of this problem? Is it contained in the nature of mathematics itself? Mathematics has been successfully applied first in the natural sciences (astronomy, physics, later chemistry, meteorology, biology), and these applications are still fruitful. This is probably the reason why mathematics is often looked upon as one of the natural sciences. But it does not belong there. Mathematical applications in economics are important today, and mathematics is an indispensible tool in any technical science. Already these applications make it impossible to view mathematics as one of the natural sciences. But there is another and fundamentally more important reason not to classify mathematics as a natural science. Its development on a superficial level is driven by needs which make themselves felt in technology and other sciences but on a deeper level by curiosity and an urge to act similar to the driving forces one finds in art. And once we realize that, this idea will have a great impact on how we plan education for different age groups, from the youngest children to postgraduate education. The first point of attention in this paper is that the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it. Second, it is just this uncanny usefulness of mathematical concepts that raises the question of the uniqueness of our physical theories. In order to establish the first point, that mathematics plays an unreasonably important role in physics, it will be useful to say a few words on the question, “What is mathematics?”, then, “What is physics?”, then, how mathematics enters physical theories, and last, why the success of mathematics in its role in physics appears so baffling. This Research in mathematics education has primarily two purposes: first to create a better understanding of the nature of mathematical thinking, teaching, and learning; and second to use such knowledge in practice for learning and teaching mathematics. It is generally acknowledged that mathematics education has a social and political dimension (e.g. the importance of mathematics in society; the relevance of mathematics to other subjects; inclusion and exclusion in terms of gender, race and social class). Moreover, mathematics education as a research domain comprises also other educational sciences and disciplines such as sociology, psychology, anthropology, linguistics, philosophy, and more recently also neuroscience.

KEYWORDS- Uniqueness theories of physics, Usefulness of mathematics, Dynamics of mathematics, Mathematics in physical theories.

1. INTRODUCTION

Mathematics is more than just the science of numbers taught by teachers in schools and either enjoyed or feared by many students. It plays a significant role in the lives of individuals and the world of society as a whole. Mathematics is an essential discipline recognized worldwide, and it needs to be augmented in education to equip students with skills necessary for achieving higher education, career aspirations, and for attaining personal fulfillment. Its significance to education is not limited to the following aspects. Mathematics enhances students' logical, functional and aesthetic skills. Problems enable students to apply their skills to both familiar and unfamiliar situations, thereby giving them the ability to use tested theory and also create their own before applying them. By developing problem solving strategies, students learn to understand problems, devise plans, carry out plans, analyze and review the accuracy of their solutions. The methods involved in problem solving develop use of reasoning, careful and reasonable argument, and decision making. Mathematics is not a mere subject that prepares students for higher academic attainment or job qualification in the future. It is not all about practicing calculations in algebra, statistics and algorithms that, after all, computers are capable of doing. It is more about how it compels the human brain to formulate problems, theories and methods of solutions. It prepares children to face a variety of simple to multifaceted challenges every human being encounters on a daily basis. Irrespective of your status in life and however basic your skills are, you apply mathematics. Daily activities including the mundane things you do are reliant on how to count, add or multiply. You encounter numbers every day in memorizing phone numbers, buying groceries, cooking food, balancing a budget, paying bills, estimating gasoline consumption, measuring distance and managing your time. In the fields of business and economy, including the diverse industries existing around you, basic to complex math applications are crucial.

Anywhere in the world, mathematics is employed as a key instrument in a diversity of fields such as medicine, engineering, natural science, social science, physical science, tech science, business and commerce, etc. Application of mathematical knowledge in every field of study and industry produces new discoveries and advancement of new disciplines. All innovations introduced worldwide, every product of technology that man gets pleasure from is a byproduct of Science and Math. The ease and convenience people enjoy today from the discoveries of computers, automobiles, aircraft, household and personal gadgets would never have happened if it were not for this essential tool used in technology.

Every branch of Mathematics has distinct applications in different types of careers. The skills enhanced from practicing math such as analyzing patterns, logical thinking, problem solving and the ability to see relationships can help you prepare for your chosen career and enable you to compete for interesting and high-paying jobs against people around the globe. Even if you do not take up math-intensive courses, you have the edge to compete against other job applicants if you have a strong mathematical background, as industries are constantly evolving together with fast-paced technology. Since mathematics encompasses all aspects of human life, it is unquestionably important in education to help students and all people from all walks of life perform daily tasks efficiently and become productive, well-informed, functional, independent individuals and members of a society where Math is a fundamental component.

2. REVIEW OF RELATED LITERATURES

To illustrate the role of mathematics in science and technology, We shall discussed a few literatures as examples. Albert Einstein (1879{1955) used Riemannian geometry and tensor calculus in his general theory of relativity. These intellectual tools were not developed for the sake of physics, but much earlier in pure mathematics. They were completely ready when Einstein began to use them. Bernhard Riemann (1826{1866) had introduced what is now called Riemannian differential geometry; on a Riemannian manifold one can compute distances and one has different concepts of curvature. Carl Friedrich Gauss (1777{1855) developed a theory for surfaces where he distinguished between intrinsic and extrinsic properties, i.e., on the one hand properties that can be studied if we live inside the surface and do not know of anything else, and on the other hand those which depend on the fact that we can look at the surface as lying in an ambient space. Gauss had to confront concrete geodetic problems concerning the inner geometry of surfaces during the triangulations of the surface of the earth that where initiated during his time. He was director of the astronomical observatory in Gottingen 1807{1855 and made degree measurements himself in 1821{1824. The geoids as a fundamental surface in geodesy was introduced by him in 1828. He was inspired by the geodetic problems but went much further in his mathematical theory than was needed for their solution. With the name of Gregorio Ricci (1853{1925) we associate the tensor calculus, which makes it possible to describe quantities of various kind and how they behave under coordinate changes. Marcel Grossman (1878{1936) explained to Einstein part of Gauss' theory of surfaces [Grattan-Guinness 1994:1239]. Tensor calculus became well-known because of the fact that Einstein used it in his general theory of relativity, published in 1916.

Another example is the theory of spectral decomposition of self-adjoint operators in Hilbert space. David Hilbert (1862{1943) published in 1912 a theory for linear integral equations. It was later extended by Torsten Carleman (1892{1949) to a more general case, called singular integral equations. Carleman, whose work appeared in 1923, expressed his results not with the help of abstract Hilbert space theory but in terms of an integral equation. It had a real and symmetric kernel, which could be so unpleasant that the corresponding operator was not continuous. It was John von Neumann (1903{1957) who put all these results into an abstract and unified theory. His work appeared in 1929. The real and symmetric kernel corresponds to a self-adjoint operator in the abstract theory. In an almost miraculous way it turned out that the results about spectral decomposition of selfadjoint operators could be used as a mathematical model in quantum mechanics. Hilbert's and Carleman's investigations did not envision that goal at all. It developed that important physical quantities correspond to discontinuous operators in the mathematical model, thus vindicating Carleman's theory: the continuous operators proved insufficient. In quantum mechanics, there exist two fundamental concepts: the states and the observable quantities. The states are equivalence classes of vectors in a complex not real Hilbert space, and the observable quantities are self-adjoint operators, not necessarily continuous, that act on these vectors. Nothing in everyday life leads directly to complex numbers, and they did not appear in any physical observations, but still they turned out to be essential for the formulation of the quantum-theoretical laws.

As a third example we may take the mathematical foundations of computer science. The theory of computable functions was developed in the thirties before modern computers existed, and it was seemingly without application. Modern logic programming is built on a theorem of Jacques Herbrand (1908{1931) from the early thirties. During that decade, Alonzo Church (1903{1995) created lambda calculus. It was published in 1941 and became the basis of the functional programming languages, of which LISP from 1960 is an example. The basic principles of how computers work were developed in the forties by, among others, John von Neumann. Self-correcting codes, which are now used in digital communication all over the world, are based on Galois theory, a creation by Evariste Galois (1811{1832). (That theory is otherwise most noted in that it shows that a fifth degree equation cannot be solved by radicals.) Can string theory become a fourth example? It exploits very modern and abstract mathematics; at the same time it inspires development of even more mathematics. It implies large changes in our view of the world. Our concept of space time seems according to Edward Witten (b. 1951) destined to turn out to be only an approximate, derived notion, much as classical concepts such as the position and velocity of a particle are understood as approximate concepts in the light of quantum mechanics" [1996:28]. It might be too early to say anything definite about the role of mathematics in this case, since string theory presently is in what some call the second superstring revolution (the first happened in the eighties); Witten [1996:30]. At least it is clear that classical mathematical concepts like manifolds and differential forms play a basic role, and that the latest development in mathematical fields such as topology and knot theory are highly relevant for what some with perhaps not fully developed humbleness call the Theory of Everything; Taubes [1995]. Let us quote Freeman Dyson (b. 1923): One factor that has remained constant through all the twists and turns of the history of physical science is the decisive importance of mathematical imagination. Each century had its own particular preoccupations in science and its own particular style in mathematics. But in every century in which major advances were achieved the growth in physical understanding was guided by a combination of empirical observation with purely mathematical intuition. For a physicist mathematics is not just a tool by means of which phenomena can be calculated; it is the main source of concepts and principles by means of which new theories can be created" [1968:249]. But of course the mathematicians are not always successful. According to Dyson [1972] it has happened a number of times that the mathematicians have missed opportunities to develop their science. For example, the equations that James Clerk Maxwell (1831{1879) published in 1873 offered a very interesting field that did not attract as much attention by the mathematicians as it deserved. Perhaps, if the mathematicians had begun to study these new problems when they first arose, they would have had the opportunity to discover relativity theory several decades before Einstein did. Dyson bases this bold statement on the fact that Maxwell's equations are invariant under certain transformations that form a group, i.e., a set consisting of transformations that can be composed and inverted. Such a group is an important mathematical object in itself. Maxwell's equations are invariant under the Lorentz group, whereas Newtonian mechanics is invariant under another group, the so-called Galilei group. The Lorentz group is mathematically simpler and more beautiful than the Galilei group. If the mathematical properties of these groups had been studied, perhaps the special theory of relativity could have been discovered. Of course we must be aware that this reasoning is in the conditional mood. We cannot prove what would have happened if the mathematicians had done something else than they did. But Dyson's speculation still points in the same direction as the preceding positive examples: a great confidence in the possibilities of finding physically interesting theories within mathematics.

Eugene Wigner (1902{1995) wrote that the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it" [1960:2]. And it is just this uncanny usefulness of mathematical concepts that raises the question of the uniqueness of our physical theories"; i.e., whether other totally different theories could explain the phenomena as well as those that we happen to have at hand. Reacting on these suggestions of the power of mathematics to influence how science is formulated, we are drawn to ask: are the theories of physics just those that the mathematical theories and methods allow a certain investigator at a certain moment? If the answer is yes, why are these methods available at a given moment? If mathematics were different, would also physics be different? What are the implications of these questions for the responsibility of the mathematicians? And what are the implications for research policy?

Mathematics is the foundation of science and technology and the functional role of mathematics to science and technology is multifaceted and multifarious that no area of science, technology and business enterprise escapes its application (Okereke, 2006). Ukeje (1986) described mathematics as the mirror of civilization in all the centuries of painstaking calculation, and the most basic discipline for any person who would be truly educated in any science and in many other endeavours. Despite the importance placed on mathematics, researchers (Odili, 1986; Salau, 1995; Amazigo, 2000; Agw- agah, 2001; Betiku, 2001; Obioma, 2005; Maduabum and Odili, 2006; Okereke, 2006) had observed that students lack interest in the subject and perform poorly in it. Ukeje (1986) observed that mathematics is one of the most poorly taught, widely hated and abysmally understood subject in secondary school, students particularly girls run away from the subject. The West African Examination Council (WAEC) Chief Examiners [2003, 2004, 2005, and 2006] consistently reported candidates' lack of skill in answering almost all the questions asked in general mathematics. WAEC Chief Examiners [2003, 2005] further observed that candidates were weak in Geometry of circles and 3- dimensional problems. According to their reports, most candidates avoided questions on 3-dimensional problem, when they attempt geometry questions; only few of the candidates showed a clear understanding of the problem in their working. WAEC [2004] also reported candidates' weakness in Algebraic expression and word problems among others. Obioma (1985), Obodo (1993) and Okereke (2006) reported gender as a significant factor in mathematics achievement and Onwioduokit and Akinbobola (2005) reported it as a significant factor in physics achievement when physics students are taught with advance organizers. However Okonkwo (1997) reported gender as non significant when students are taught with tangram puzzle game. Okereke (2006) attributed students' poor performance to factors such as the society view that mathematics is difficult, shortage of qualified teachers, and lack of incentive (motivation). The abstract nature of mathematics should be reduced through demonstration and practical methods. Agwagah (1997) observed that the problem of ineffective teaching can be tackled through planned and intelligent application of the mathematics. The method of drill and verbal recitation makes learning boring and lacks motivation for further learning. It is important therefore to consider strategies that may help to improve the performance, with the view of considering their effect on teaching and learning of mathematics.

3. THE DYNAMICS OF MATHEMATICS

Many people believe that mathematics is a collection of fixed truths and unchangeable laws. It is not hard to see the roots of such a belief. We learn that two plus two is four, and we cannot imagine that this truth one day should be untrue. A stone that we see on the ground can be several billions of years old, and it might be dust within a few million years, but at that time it will still be true that two plus two is four. Or don't you believe that? Mathematics appears to be much more stable than the most stable parts of our physical reality. Even general knowledge has changed more in other fields.

According to the theory that Alfred Wegener (1880{1930) published in 1912, the continents are moving relative to each other. When I went to school I learned that his theory was naive and false. We school children nevertheless thought that Africa and South America fit rather well together. Nowadays it is an established fact that these continents once were together. I also learned that humans have 48 chromosomes. Now the children learn that a human has 46 chromosomes. (The number 48 is said to come from a miscalculation that was done on a photo where everyone today sees only 46.) In this way my knowledge about the world has changed. On the other hand, in mathematics I learned more than twenty years ago that the derivative of the function $x \to x^4$ is $x \to 4x^3$, and so far I have not heard anything else. These facts give an inevitable impression that the geosciences develop, biology develops, but not mathematics. Or is that impression really inevitable?

I claim that mathematics, like a living creature, consists of immobile and dynamic parts. Does a human need rigid bones or soft muscles? To be able to run, it seems that a human needs both. The skeleton alone cannot move, and without it the muscles have nothing to work against. Similarly, while some parts of mathematics appear to be very immobile, others are in a state of fast development and very dynamics. The parts which have been immobile for a long time are what we teach in schools; the dynamic parts are less known. Thus it may not be so surprising that people think of mathematics more as a skeleton rather than as muscles. Every year tens of thousands of articles are published about new results in mathematics. Lots of new facts become known and old ones become understood in a new light. (And... it might be added here that mathematics is free of the often very hampering difficulties of experiments or observations that hold up experimental sciences...)

But it is not only that mathematics develops: mathematics also contains a lot of arbitrariness. In the same way that the rigid mathematics is extremely stable, the mobile mathematics is extremely dynamic in its unlimited arbitrariness. This can be very disturbing for those who resort to mathematics in a desire for security and stable values; the arbitrariness makes them disappointed and even can seem scary.

One example of this arbitrariness comes from the history of the parallel postulate. According to this axiom posed by Euclid (ca. 303 { ca. 275 B.C.), there is exactly one straight line through a given point that is parallel to a given straight line. Is it possible to prove this axiom using the other axioms? This question occupied mathematicians for two millenia. Finally three mathematicians in the nineteenth century proved that this is impossible. They were Janos Bolyai (1802{1860), Nikolaj Ivanovich Lobachevskij (b. 1792 or 1793, d. 1856) and Gauss. They proved this by constructing geometries where through a given point there is either none or more than one straight line. And these geometries are as valid and as true as Euclid's. Through the existence of these new geometries, in which all other axioms are valid, one understands that Euclid's parallel postulate is not possible to prove by means of the other axioms. Because if that were the case, the parallel postulate would also be true in the new geometries. Elementary! Why did the solution of this problem take two thousand years? Such a question can hardly be answered, but a possible reason is that it was very shocking for people to accept the fact of the arbitrariness of the axioms, these so-called self-evident" starting-points for the human mind. Such an explanation is supported by the fact that Gauss did not publish his discovery despite the fact that he was highly respected and would not have risked his career by publishing it.

Another example of arbitrariness is perhaps even more dramatic when it comes to the limits of our reasoning power: the independence of the continuum hypothesis. This hypothesis says that every infinite subset of the field R of real numbers (in this context called the continuum) either has as many elements as the natural numbers N or as the whole continuum R. To express this with mathematical symbols we shall denote the number of element in a set A by card A; we say that card A is the cardinal number of the set A. It is a number, finite or infinite. (As an example we mention that prime numbers have the same cardinal number or cardinality as N, as do the rational numbers, whereas the positive numbers have the same cardinality as the whole continuum.) The continuum hypothesis says that it cannot happen that $\text{card } N < \text{card } A < \text{card } R$. The proof of this was the first of twenty three problems posed by Hilbert in Paris in 1900 as the future problems of mathematics." He thought it was very plausible that this conjecture was true [1902:70].

To Hilbert, like probably to any mathematician of his generation, either there existed a subset A of R such that card N < card A < card R, or there did not exist such a set. Research should make it clear to us which alternative was the right one. But it later turned out that the continuum hypothesis is independent of the other axioms. According to Kurt Godel (1906{1978) one can add the continuum hypothesis to the other axioms of set theory without introducing (new) contradictions, and according to Paul Cohen (b. 1934) one can do the same with the negation of the hypothesis. This means that a set theory where there exists a set A with card N < card A < card R is as valid and as true as a set theory where the continuum hypothesis is valid.

To sum up we can say that mathematics does not help us to verify whether, in the real world, there is no, one or many straight lines through a given point parallel with a given straight line. Also mathematics does not help us to verify whether there exist or does not exist certain infinite sets. Here the arbitrariness of mathematics manifests itself, and it leaves us in the lurch. But at the same time, paradoxically, we should remember that mathematics is the main or even only source of concepts and principles in the natural sciences, and the only language in which the natural sciences can express derivations and results.

4. METHODOLOGY

There is a story about two friends, who were classmates in high school, talking about their jobs. One of them became a statistician and was working on population trends. He showed a reprint to his former classmate. The reprint started, as usual, with the Gaussian distribution and the statistician explained to his former classmate the meaning of the symbols for the actual population, for the average population, and so on. His classmate was a bit incredulous and was not quite sure whether the statistician was pulling his leg. "How can you know that?" was his query. "And what is this symbol here?" "Oh," said the statistician, "this is pi." "What is that?" "The ratio of the circumference of the circle to its diameter." "Well, now you are pushing your joke too far," said the classmate, "surely the population has nothing to do with the circumference of the circle." Naturally, we are inclined to smile about the simplicity of the classmate's approach. Nevertheless, when I heard this story, I had to admit to an eerie feeling because, surely, the reaction of the classmate betrayed only plain common sense. I was even more confused when, not many days later, someone came to me and expressed his bewilderment with the fact that we make a rather narrow selection when choosing the data on which we test our theories. "How do we know that, if we made a theory which focuses its attention on phenomena we disregard and disregards some of the phenomena now commanding our attention, that we could not build another theory which has little in common with the present one but which, nevertheless, explains just as many phenomena as the present theory?"

It has to be admitted that we have no definite evidence that there is no such theory. The preceding two stories illustrate the two main points which are the subjects of the present discourse. The first point is that mathematical concepts turn up in entirely unexpected connections. Moreover, they often permit an unexpectedly close and accurate description of the phenomena in these connections. Secondly, just because of this circumstance, and because we do not understand the reasons of their usefulness, we cannot know whether a theory formulated in terms of mathematical concepts is uniquely appropriate. We are in a position similar to that of a man who was provided with a bunch of keys and who, having to open several doors in succession, always hit on the right key on the first or second trial. He became skeptical concerning the uniqueness of the coordination between keys and doors. Most of what will be said on these questions will not be new; it has probably occurred to most scientists in one form or another. My principal aim is to illuminate it from several sides. The first point is that the enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it. Second, it is just this uncanny usefulness of mathematical concepts that raises the question of the uniqueness of our physical theories. In order to establish the first point, that mathematics plays an unreasonably important role in physics, it will be useful to say a few words on the question, "What is mathematics?", then, "What is physics?", then, how mathematics enters physical theories, and last, why the success of mathematics in its role in physics appears so baffling. Much less will be said on the second point: the uniqueness of the theories of physics. A proper answer to this question would require elaborate experimental and theoretical work which has not been undertaken to date.

5. RESULTS AND DISCUSSION

Somebody once said that philosophy is the misuse of a terminology which was invented just for this purpose. In the same vein, I would say that mathematics is the science of skillful operations with concepts and rules invented just for this purpose. The principal emphasis is on the invention of concepts. Mathematics would soon run out of interesting theorems if these had to be formulated in terms of the concepts which already appear in the axioms. Furthermore, whereas it is unquestionably true that the concepts of elementary mathematics and particularly elementary geometry were formulated to describe entities which are directly suggested by the actual world, the same does not seem to be true of the more advanced concepts, in particular the concepts which play such an important role in physics. Thus, the rules for operations with pairs of numbers are obviously designed to give the same results as the operations with fractions which we first learned without reference to "pairs of numbers." The rules for the operations with sequences, that is, with irrational numbers, still belong to the category of rules which were determined so as to reproduce rules for the operations with quantities which were already known to us. Most more advanced mathematical concepts, such as complex numbers, algebras, linear operators, Borel sets, and this list could be continued almost indefinitely, were so devised that they are apt subjects on which the mathematician can demonstrate his ingenuity and sense of formal beauty. In fact, the definition of these concepts, with a realization that interesting and ingenious considerations could be applied to them, is the first demonstration of the ingeniousness of the mathematician who defines them. The depth of thought which goes into the formulation of the mathematical concepts is later justified by the skill with which these concepts are used. The great mathematician fully, almost ruthlessly, exploits the domain of permissible reasoning and skirts the impermissible. That his recklessness does not lead him into a morass of contradictions is a miracle in itself: certainly it is hard to believe that our reasoning power was brought, by Darwin's process of natural selection, to the perfection which it seems to possess. However, this is not our present subject. The principal point which will have to be recalled later is that the mathematician could formulate only a handful of interesting theorems without defining concepts beyond those contained in the axioms and that the concepts outside those contained in the axioms are defined with a view of permitting ingenious logical operations which appeal to our aesthetic sense both as operations and also in their results of great generality and simplicity. The complex numbers provide a particularly striking example for the foregoing. Certainly, nothing in our experience suggests the introduction of these quantities. Indeed, if a mathematician is asked to justify his interest in complex numbers, he will point, with some indignation, to the many beautiful theorems in the theory of equations, of power series, and of analytic functions in general, which owe their origin to the introduction of complex numbers. The mathematician is not willing to give up his interest in these most beautiful accomplishments.

The physicist is interested in discovering the laws of inanimate nature. In order to understand this statement, it is necessary to analyze the concept, "law of nature." The world around us is of baffling complexity and the most obvious fact about it is that we cannot predict the future. Although the joke attributes only to the optimist the view that the future is uncertain, the optimist is right in this case: the future is unpredictable. It is, as Schrodinger has remarked, a miracle that in spite of the baffling complexity of the world, certain regularities in the events could be discovered. One such regularity, discovered by Galileo, is that two rocks, dropped at the same time from the same height, reach the ground at the same time. The laws of nature are concerned with such regularities. Galileo's regularity is a prototype of a large class of regularities. It is a surprising regularity for three reasons.

The first reason that it is surprising is that it is true not only in Pisa, and in Galileo's time, it is true everywhere on the Earth, was always true, and will always be true. This property of the regularity is a recognized invariance property and, as I had occasion to point out some time ago, without invariance principles similar to those implied in the preceding generalization of Galileo's observation, physics would not be possible. The second surprising feature is that the regularity which we are discussing is independent of so many conditions which could have an effect on it. It is valid no matter whether it rains or not, whether the experiment is carried out in a room or from the Leaning Tower, no matter whether the person who drops the rocks is a man or a woman. It is valid even if the two rocks are dropped, simultaneously and from the same height, by two different people. There are, obviously, innumerable other conditions which are all immaterial from the point of view of the validity of Galileo's regularity. The irrelevancy of so many circumstances which could play a role in the phenomenon observed has also been called an invariance. However, this invariance is of a different character from the preceding one since it cannot be formulated as a general principle. The exploration of the conditions which do, and which do not, influence a phenomenon is part of the early experimental exploration of a field. It is the skill and ingenuity of the experimenter which show him phenomena which depend on a relatively narrow set of relatively easily realizable and reproducible conditions.

In the present case, Galileo's restriction of his observations to relatively heavy bodies was the most important step in this regard. Again, it is true that if there were no phenomena which are independent of all but a manageably small set of conditions, physics would be impossible.

The preceding two points, though highly significant from the point of view of the philosopher, are not the ones which surprised Galileo most, nor do they contain a specific law of nature. The law of nature is contained in the statement that the length of time which it takes for a heavy object to fall from a given height is independent of the size, material, and shape of the body which drops. In the framework of Newton's second "law," this amounts to the statement that the gravitational force which acts on the falling body is proportional to its mass but independent of the size, material, and shape of the body which falls.

The preceding discussion is intended to remind us, first, that it is not at all natural that "laws of nature" exist, much less that man is able to discover them. The present writer had occasion, some time ago, to call attention to the succession of layers of "laws of nature," each layer containing more general and more encompassing laws than the previous one and its discovery constituting a deeper penetration into the structure of the universe than the layers recognized before. However, the point which is most significant in the present context is that all these laws of nature contain, in even their remotest consequences, only a small part of our knowledge of the inanimate world. All the laws of nature are conditional statements which permit a prediction of some future events on the basis of the knowledge of the present, except that some aspects of the present state of the world, in practice the overwhelming majority of the determinants of the present state of the world, are irrelevant from the point of view of the prediction. The irrelevancy is meant in the sense of the second point in the discussion of Galileo's theorem.

As regards the present state of the world, such as the existence of the earth on which we live and on which Galileo's experiments were performed, the existence of the sun and of all our surroundings, the laws of nature are entirely silent. It is in consonance with this, first, that the laws of nature can be used to predict future events only under exceptional circumstances when all the relevant determinants of the present state of the world are known. It is also in consonance with this that the construction of machines, the functioning of which he can foresee, constitutes the most spectacular accomplishment of the physicist. In these machines, the physicist creates a situation in which all the relevant coordinates are known so that the behavior of the machine can be predicted. Radars and nuclear reactors are examples of such machines.

The principal purpose of the preceding discussion is to point out that the laws of nature are all conditional statements and they relate only to a very small part of our knowledge of the world. Thus, classical mechanics, which is the best known prototype of a physical theory, gives the second derivatives of the positional coordinates of all bodies, on the basis of the knowledge of the positions, etc., of these bodies. It gives no information on the existence, the present positions, or velocities of these bodies. It should be mentioned, for the sake of accuracy, that we discovered about twenty years ago that even the conditional statements cannot be entirely precise: that the conditional statements are probability laws which enable us only to place intelligent bets on future properties of the inanimate world, based on the knowledge of the present state. They do not allow us to make categorical statements, not even categorical statements conditional on the present state of the world. The probabilistic nature of the "laws of nature" manifests itself in the case of machines also, and can be verified, at least in the case of nuclear reactors, if one runs them at very low power. However, the additional limitation of the scope of the laws of nature which follows from their probabilistic nature will play no role in the rest of the discussion.

5.1 The Place of Mathematics in Physical Theories

Having refreshed our minds as to the essence of mathematics and physics, we should be in a better position to review the role of mathematics in physical theories. Naturally, we do use mathematics in everyday physics to evaluate the results of the laws of nature, to apply the conditional statements to the particular conditions which happen to prevail or happen to interest us. In order that this be possible, the laws of nature must already be formulated in mathematical language. However, the role of evaluating the consequences of already established theories is not the most important role of mathematics in physics. Mathematics, or, rather, applied mathematics, is not so much the master of the situation in this function: it is merely serving as a tool.

Mathematics does play, however, also a more sovereign role in physics. This was already implied in the statement, made when discussing the role of applied mathematics, that the laws of nature must have been formulated in the language of mathematics to be an object for the use of applied mathematics. The statement that the laws of nature are written in the language of mathematics was properly made three hundred years ago; it is now more true than ever before. In order to show the importance which mathematical concepts possess in the formulation of the laws of physics, let us recall, as an example, the axioms of quantum mechanics as formulated, explicitly, by the great physicist, Dirac. There are two basic concepts in quantum mechanics: states and observables. The states are vectors in Hilbert space, the observables self-adjoint operators on these vectors. The possible values of the observations are the characteristic values of the operators but we had better stop here lest we engage in a listing of the mathematical concepts developed in the theory of linear operators.

It is true, of course, that physics chooses certain mathematical concepts for the formulation of the laws of nature, and surely only a fraction of all mathematical concepts is used in physics. It is true also that the concepts which were chosen were not selected arbitrarily from a listing of mathematical terms but were developed, in many if not most cases, independently by the physicist and recognized then as having been conceived before by the mathematician. It is not true, however, as is so often stated, that this had to happen because mathematics uses the simplest possible concepts and these were bound to occur in any formalism. As we saw before, the concepts of mathematics are not chosen for their conceptual simplicity, even sequences of pairs of numbers are far from being the simplest concepts, but for their amenability to clever manipulations and to striking, brilliant arguments. Let us not forget that the Hilbert space of quantum mechanics is the complex Hilbert space, with a Hermitean scalar product. Surely to the unpreoccupied mind, complex numbers are far from natural or simple and they cannot be suggested by physical observations. Furthermore, the use of complex numbers is in this case not a calculational trick of applied mathematics but comes close to being a necessity in the formulation of the laws of quantum mechanics. Finally, it now begins to appear that not only complex numbers but so-called analytic functions are destined to play a decisive role in the formulation of quantum theory. I am referring to the rapidly developing theory of dispersion relations.

It is difficult to avoid the impression that a miracle confronts us here, quite comparable in its striking nature to the miracle that the human mind can string a thousand arguments together without getting itself into contradictions, or to the two miracles of the existence of laws of nature and of the human mind's capacity to divine them. The observation which comes closest to an explanation for the mathematical concepts' cropping up in physics which I know is Einstein's statement that the only physical theories which we are willing to accept are the beautiful ones. It stands to argue that the concepts of mathematics, which invite the exercise of so much wit, have the quality of beauty. However, Einstein's observation can at best explain properties of theories which we are willing to believe and has no reference to the intrinsic accuracy of the theory. We shall, therefore, turn to this latter question. Is the Success of Physical Theories Truly Surprising?
A possible explanation of the physicist's use of mathematics to formulate his laws of nature is that he is a somewhat irresponsible person. As a result, when he finds a connection between two quantities which resembles a connection well-known from mathematics, he will jump at the conclusion that the connection is that discussed in mathematics simply because he does not know of any other similar connection. It is not the intention of the present discussion to refute the charge that the physicist is a somewhat irresponsible person. Perhaps he is. However, it is important to point out that the mathematical formulation of the physicist's often crude experience leads in an uncanny number of cases to an amazingly accurate description of a large class of phenomena. This shows that the mathematical language has more to commend it than being the only language which we can speak; it shows that it is, in a very real sense, the correct language. Let us consider a few examples.

The first example is the oft-quoted one of planetary motion. The laws of falling bodies became rather well established as a result of experiments carried out principally in Italy. These experiments could not be very accurate in the sense in which we understand accuracy today partly because of the effect of air resistance and partly because of the impossibility, at that time, to measure short time intervals. Nevertheless, it is not surprising that, as a result of their studies, the Italian natural scientists acquired a familiarity with the ways in which objects travel through the atmosphere. It was Newton who then brought the law of freely falling objects into relation with the motion of the moon, noted that the parabola of the thrown rock's path on the earth and the circle of the moon's path in the sky are particular cases of the same mathematical object of an ellipse, and postulated the universal law of gravitation on the basis of a single, and at that time very approximate, numerical coincidence. Philosophically, the law of gravitation as formulated by Newton was repugnant to his time and to himself. Empirically, it was based on very scanty observations. The mathematical language in which it was formulated contained the concept of a second derivative and those of us who have tried to draw an osculating circle to a curve know that the second derivative is not a very immediate concept. The law of gravity which Newton reluctantly established and which he could verify with an accuracy of about 4% has proved to be accurate to less than a ten thousandth of a per cent and became so closely associated with the idea of absolute accuracy that only recently did physicists become again bold enough to inquire into the limitations of its accuracy. Certainly, the example of Newton's law, quoted over and over again, must be mentioned first as a monumental example of a law, formulated in terms which appear simple to the mathematician, which has proved accurate beyond all reasonable expectations. Let us just recapitulate our thesis on this example: first, the law, particularly since a second derivative appears in it, is simple only to the mathematician, not to common sense or to non-mathematically-minded freshmen; second, it is a conditional law of very limited scope. It explains nothing about the earth which attracts Galileo's rocks, or about the circular form of the moon's orbit, or about the planets of the sun. The explanation of these initial conditions is left to the geologist and the astronomer, and they have a hard time with them. The second example is that of ordinary, elementary quantum mechanics. This originated when Max Born noticed that some rules of computation, given by Heisenberg, were formally identical with the rules of computation with matrices, established a long time before by mathematicians.

Born, Jordan, and Heisenberg then proposed to replace by matrices the position and momentum variables of the equations of classical mechanics. They applied the rules of matrix mechanics to a few highly idealized problems and the results were quite satisfactory. However, there was, at that time, no rational evidence that their matrix mechanics would prove correct under more realistic conditions. Indeed, they say "if the mechanics as here proposed should already be correct in its essential traits." As a matter of fact, the first application of their mechanics to a realistic problem, that of the hydrogen atom, was given several months later, by Pauli. This application gave results in agreement with experience. This was satisfactory but still understandable because Heisenberg's rules of calculation were abstracted from problems which included the old theory of the hydrogen atom. The miracle occurred only when matrix mechanics, or a mathematically equivalent theory, was applied to problems for which Heisenberg's calculating rules were meaningless. Heisenberg's rules presupposed that the classical equations of motion had solutions with certain periodicity properties; and the equations of motion of the two electrons of the helium atom, or of the even greater number of electrons of heavier atoms, simply do not have these properties, so that Heisenberg's rules cannot be applied to these cases. Nevertheless, the calculation of the lowest energy level of helium, as carried out a few months ago by Kinoshita at Cornell and by Bazley at the Bureau of Standards, agrees with the experimental data within the accuracy of the observations, which is one part in ten million. Surely in this case we "got something out" of the equations that we did not put in.

The same is true of the qualitative characteristics of the "complex spectra," that is, the spectra of heavier atoms. I wish to recall a conversation with my Graduate degree supervisor Dr Musa of Bayero University Kano Nigeria, who told me, when the qualitative features of the spectra were derived, that a disagreement of the rules derived from quantum mechanical theory and the rules established by empirical research would have provided the last opportunity to make a change in the framework of matrix mechanics. In other words, Dr Musa felt that I would have been, at least temporarily, helpless had an unexpected disagreement occurred in the theory of the helium atom. This was, at that time, developed by Kellner and by Hilleraas. The mathematical formalism was too dear and unchangeable so that, had the miracle of helium which was mentioned before not occurred, a true crisis would have arisen. Surely, physics would have overcome that crisis in one way or another. It is true, on the other hand, that physics as we know it today would not be possible without a constant recurrence of miracles similar to the one of the helium atom, which is perhaps the most striking miracle that has occurred in the course of the development of elementary quantum mechanics, but by far not the only one. In fact, the number of analogous miracles is limited, in my view, only by our willingness to go after more similar ones. Quantum mechanics had, nevertheless, many almost equally striking successes which gave us the firm conviction that it is, what we call, correct.

The last example is that of quantum electrodynamics, or the theory of the Lamb shift. Whereas Newton's theory of gravitation still had obvious connections with experience, experience entered the formulation of matrix mechanics only in the refined or sublimated form of Heisenberg's prescriptions. The quantum theory of the Lamb shift, as conceived by Bethe and established by Schwinger, is a purely mathematical theory and the only direct contribution of experiment was to show the existence of a measurable effect. The agreement with calculation is better than one part in a thousand.

The preceding three examples, which could be multiplied almost indefinitely, should illustrate the appropriateness and accuracy of the mathematical formulation of the laws of nature in terms of concepts chosen for their manipulability, the "laws of nature" being of almost fantastic accuracy but of strictly limited scope. I propose to refer to the observation which these examples illustrate as the empirical law of epistemology. Together with the laws of invariance of physical theories, it is an indispensable foundation of these theories. Without the laws of invariance the physical theories could have been given no foundation of fact; if the empirical law of epistemology were not correct, we would lack the encouragement and reassurance which are emotional necessities, without which the "laws of nature" could not have been successfully explored. Dr. Musa, with whom I discussed the empirical law of epistemology, called it an article of faith of the theoretical physicist, and it is surely that. However, what he called our article of faith can be well supported by actual examples, many examples in addition to the three which have been mentioned.

The empirical nature of the preceding observation seems to me to be self-evident. It surely is not a "necessity of thought" and it should not be necessary, in order to prove this, to point to the fact that it applies only to a very small part of our knowledge of the inanimate world. It is absurd to believe that the existence of mathematically simple expressions for the second derivative of the position is self-evident, when no similar expressions for the position itself or for the velocity exist. It is therefore surprising how readily the wonderful gift contained in the empirical law of epistemology was taken for granted. The ability of the human mind to form a string of 1000 conclusions and still remain "right," which was mentioned before, is a similar gift. Every empirical law has the disquieting quality that one does not know its limitations. We have seen that there are regularities in the events in the world around us which can be formulated in terms of mathematical concepts with an uncanny accuracy. There are, on the other hand, aspects of the world concerning which we do not believe in the existence of any accurate regularities. We call these initial conditions. The question which presents itself is whether the different regularities, that is, the various laws of nature which will be discovered, will fuse into a single consistent unit, or at least asymptotically approach such a fusion. Alternatively, it is possible that there always will be some laws of nature which have nothing in common with each other.

At present, this is true, for instance, of the laws of heredity and of physics. It is even possible that some of the laws of nature will be in conflict with each other in their implications, but each convincing enough in its own domain so that we may not be willing to abandon any of them. We may resign ourselves to such a state of affairs or our interest in clearing up the conflict between the various theories may fade out. We may lose interest in the "ultimate truth," that is, in a picture which is a consistent fusion into a single unit of the little pictures, formed on the various aspects of nature. It may be useful to illustrate the alternatives by an example. We now have, in physics, two theories of great power and interest: the theory of quantum phenomena and the theory of relativity. These two theories have their roots in mutually exclusive groups of phenomena.

Relativity theory applies to macroscopic bodies, such as stars. The event of coincidence, that is, in ultimate analysis of collision, is the primitive event in the theory of relativity and defines a point in space-time, or at least would define a point if the colliding panicles were infinitely small. Quantum theory has its roots in the microscopic world and, from its point of view, the event of coincidence, or of collision, even if it takes place between particles of no spatial extent, is not primitive and not at all sharply isolated in space-time. The two theories operate with different mathematical concepts, the four dimensional Riemann space and the infinite dimensional Hilbert space, respectively. So far, the two theories could not be united, that is, no mathematical formulation exists to which both of these theories are approximations. All physicists believe that a union of the two theories is inherently possible and that we shall find it. Nevertheless, it is possible also to imagine that no union of the two theories can be found. This example illustrates the two possibilities, of union and of conflict, mentioned before, both of which are conceivable. In order to obtain an indication as to which alternative to expect ultimately, we can pretend to be a little more ignorant than we are and place ourselves at a lower level of knowledge than we actually possess. If we can find a fusion of our theories on this lower level of intelligence, we can confidently expect that we will find a fusion of our theories also at our real level of intelligence. On the other hand, if we would arrive at mutually contradictory theories at a somewhat lower level of knowledge, the possibility of the permanence of conflicting theories cannot be excluded for ourselves either. The level of knowledge and ingenuity is a continuous variable and it is unlikely that a relatively small variation of this continuous variable changes the attainable picture of the world from inconsistent to consistent. Considered from this point of view, the fact that some of the theories which we know to be false give such amazingly accurate results is an adverse factor. Had we somewhat less knowledge, the group of phenomenon which these “false” theories explain would appear to us to be large enough to “prove” these theories.

However, these theories are considered to be “false” by us just for the reason that they are, in ultimate analysis, incompatible with more encompassing pictures and, if sufficiently many such false theories are discovered, they are bound to prove also to be in conflict with each other.

Similarly, it is possible that the theories, which we consider to be “proved” by a number of numerical agreements which appears to be large enough for us, are false because they are in conflict with a possible more encompassing theory which is beyond our means of discovery. If this were true, we would have to expect conflicts between our theories as soon as their number grows beyond a certain point and as soon as they cover a sufficiently large number of groups of phenomena. In contrast to the article of faith of the theoretical physicist mentioned before, this is the nightmare of the theorist.

Let us consider a few examples of “false” theories which give, in view of their falseness, alarmingly accurate descriptions of groups of phenomena. With some goodwill, one can dismiss some of the evidence which these examples provide. The success of Bohr’s early and pioneering ideas on the atom was always a rather narrow one and the same applies to Ptolemy’s epicycles. Our present vantage point gives an accurate description of all phenomena which these more primitive theories can describe. The same is not true any longer of the so-called free-electron theory, which gives a marvellously accurate picture of many, if not most, properties of metals, semiconductors, and insulators. In particular, it explains the fact, never properly understood on the basis of the “real theory,” that insulators show a specific resistance to electricity which may be 1026 times greater than that of metals. In fact, there is no experimental evidence to show that resistance is not infinite under the conditions under which the free-electron theory would lead us to expect an infinite resistance. Nevertheless, we are convinced that the free-electron theory is a crude approximation which should be replaced, in the description of all phenomena concerning solids, by a more accurate picture. If viewed from our real vantage point, the situation presented by the free-electron theory is irritating but is not likely to forebode any inconsistencies which are insurmountable for us.

The free-electron theory raises doubts as to how much we should trust numerical agreement between theory and experiment as evidence for the correctness of the theory. We are used to such doubts. A much more difficult and confusing situation would arise if we could, some day, establish a theory of the phenomena of consciousness, or of biology, which would be as coherent and convincing as our present theories of the inanimate world. Mendel’s laws of inheritance and the subsequent work on genes may well form the beginning of such a theory as far as biology is concerned. Furthermore,, it is quite possible that an abstract argument can be found which shows that there is a conflict between such a theory and the accepted principles of physics. The argument could be of such abstract nature that it might not be possible to resolve the conflict, in favor of one or of the other theory, by an experiment. Such a situation would put a heavy strain on our faith in our theories and on our belief in the reality of the concepts which we form. It would give us a deep sense of frustration in our search for what I called “the ultimate truth.”

The reason that such a situation is conceivable is that, fundamentally, we do not know why our theories work so well. Hence, their accuracy may not prove their truth and consistency. Indeed, it is this author’s belief that something rather akin to the situation which was described above exists if the present laws of heredity and of physics are confronted.

6. RECOMMENDATION

Finally, what is the cultural significance of mathematics? The answer definitely depends on ones own values. Here I limit myself to four properties which I think mathematics holds compared to other cultural phenomena:

> Internationality
> Beauty
> Influence on our view of the world
> Influence on our own thought processes and our confidence in them

When it comes to internationality it has to be said that there does not exist anything absolutely international. But a cultural phenomenon can be more or less varying within mankind. And mathematics as a subculture is certainly more international than many other cultural phenomena, and also more than many other sciences, in particular the social sciences. Mathematics as subculture can influence education in mathematics and make it more international; often that would be a good thing. But we must note also that scientific mathematics is not completely international. There are a number of national characteristics in it. We should distinguish internationality from the crossing of frontiers that is made possible by superior means of communication.

As with any cultural phenomenon we can ask: Which are the laws" according to which culture develops? What is most important? Who decides what is most important? To decide what is most important is real power. The beauty of mathematics is an essential property, and it is important from a number of viewpoints. As in the arts beauty is a value. But not only that: it is also the fastest guide in the continuous choice between different paths that a developing theory can take. Mathematics influences our view of the world; in the most mathematics sciences, no other language even seems possible. So far mathematics has mainly had an ordering function: it assures us that the world is not arbitrary and chaotic but possible to order and predict. It is a fact that the desire to be able to predict (eclipses, the weather) has been an important source for the tendency towards mathematization. But also chaos has its mathematics! Mathematics certainly governs the picture of the world that we make for Ourselves but to what extent? Mathematics also influences our mental abilities. The human brain is influenced and changed by the work it executes, at least during the first years like a human computer that builds itself while it works. The use of language and all theoretical work influence the young brain's development. Even single speech sounds shape the brain; Naatanen et al. [1997]. This certainly makes it important to choose a good occupation! If we can solve problems and avoid difficulties, then personality profits. That way mathematics can build our self-confidence (if we succeed) or destroy it (if we fail). All this points to the importance of creating a mathematical environment which is as good as possible, especially during childhood.

7. CONCLUSION

Let me end on a more cheerful note. The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning. Since the role of mathematics, as we just saw, is so paradoxical in relation to the other sciences, it is permissible, and perhaps also desirable, to look for other perspectives that can explain its function. One such alternative perspective is to accept that mathematics is part of the human culture and to compare it in general with other cultural phenomena. White [1956] and Wilder [1981] have written from this point of view. First of all we should make it clear that human culture can be of two kinds: a cultural element is a part of the culture common to a certain group of people; a subculture is a culture that is specific to a certain subgroup of that group (the subgroup is too small or too spread out to carry a culture itself).

Mathematics plays a role both as cultural element and as subculture. As part of culture, mathematics consists of all the mathematical knowledge, views and skills that a certain people own collectively. To keep these alive and perhaps expand them is a goal for general education. As an example we mention that most people are not familiar with the concepts of differentiation and integration of functions, but nevertheless have an idea of speed (in kilometers per hour), acceleration (increase of speed), interest on a mortgage, summing of monthly payments to an annual salary, as well as other things that are concrete manifestation of the abstract concepts of differentiation and integration of functions. As we see, the exact delimitation of this part of culture is not an easy task, but at least we can observe that it consists solely of parts of mathematics that were completed a long time ago. On the other hand, mathematics as subculture is the culture that is specific for people who have had training in mathematics as a science. Although this group is certainly not homogeneous, it is an interesting observation that it is more alike between one country and another than many other cultural phenomena, and in particular more alike than in school mathematics. Long ago one could talk about Chinese, Arabic, Greek and South American mathematics, but hardly any longer. If mathematics is culture.

Why would we view mathematics as culture? Normally we look upon a phenomenon as culture in order to understand it and forecast its development in that framework. I do not dare to forecast very much, but in my opinion this point of view of this paradoxical science is fruitful in order to formulate and understand many difficult problems. Dyson wrotes that science is a human activity, and the best way to understand it is to understand the individual human beings who practise it. Science is an art form and not a philosophical method" [1996:805]. Between the two concepts, mathematics as a cultural element in the culture of a whole nation and mathematics as subculture, there exists a certain tension which is visible for instance in education. Indeed, education is an introduction to the cultural element as well as to the subculture, in varying degrees from the early years to postgraduate studies. I certainly cannot scrutinize mathematics education on the whole planet, but I cannot avoid noticing that mathematics education in many countries is not successful. It is often too formal and too much concentrated on transferring routine skills.

This gives the school children an impression of mathematics being the driest and least interesting field of knowledge in the world. In psychology, one sometimes differentiates between two types of intelligence, the so-called convergent intelligence and the so-called divergent intelligence; see, e.g., Massarenti [1980]. The former is the ability to start from given conditions and reach a solution that is uniquely determined or at least the only acceptable one. The latter starts from the given situation and, along different routes, searches solutions that work, and none of which is the only acceptable one. The risk with school mathematics is that it tends to stimulate only convergent thinking, and that the given problems are so stereotyped and well prepared for treatment by routine methods that divergent intelligence seems unnecessary. It is clear that convergent intelligence is only a special case of divergent intelligence, and convergent intelligence probably has to be trained first in order to develop work methods that later can be applied to more complicated situations, where an intelligence of divergent type is needed. Of course divergent intelligence is indispensible on the research level in any science otherwise we would not be talking about research. We can schematically perhaps too schematically divide mathematics according to three criteria: cultural status, ability to change, and intelligence type required. Let us make out the divisions: Mathematics as cultural element vs. Mathematics as subculture Immobile, skeleton {like" mathematics vs. Mobile, arbitrary, muscular" mathematics Requires convergent intelligence vs. Requires divergent intelligence Could it be that these three divisions coincide, more or less? If the answer is yes, we must make an effort to change mathematics as a cultural element. I believe that general mathematics education would improve if it became more movable, less routine, and if more divergent thinking was required to solve its exercises. Why? The applications of mathematics would that way gain in quality and become more credible and more realistic. That would influence in a positive way all fields of knowledge where mathematics is used. But to change education is not an easy task, partly because people who like convergent thinking already are attracted by school mathematics and are disinclined to make it less skeleton-like."

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